Comments on Double Smoothly Undulating Integers.

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An integer can be described as *smoothly undulating* if it consists of two alternating digits, for example 12121. Clifford Pickover called an integer that is smoothly undulating in base 10 and in base 2 a *double smoothly undulating integer*, and asked if such a number exists [1]. (To prevent the question from being trivial, only numbers with more than two digits are considered.)

This article generalizes double smoothly undulating integers to numbers that are smoothly undulating in any two bases. For instance, I will consider 209 to be double smoothly undulating since it smoothly undulates in bases 9 and 13: $209_{10} = 252_9 = 131_{13}$. I examined all smoothly undulating integers of 400 digits or fewer in bases 2 to 16 and found many pairs of bases with double smoothly undulating integers (although Pickover's question remains open). Table 1 shows the results.

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Insert Table 1 about here

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The longest double smoothly undulating numbers I found all involve base 10. The number 494949 smoothly undulates in bases 10 and 15. The numbers 191919, 383838, 575757, 767676, and 959595 smoothly undulate in bases 10 and 11. Another interesting finding was many triple smoothly undulating integers, and two quadruple smoothly undulating integers: 300 smoothly undulates in bases 7, 8, 9, and 13, and 1220 smoothly undulates in bases 11, 12, 15, and 16.

For some pairs of bases, there are an infinite number of solutions. For instance, bases n and n^k have an infinite number of solutions for odd k, since k digits in the smaller base combine to form undulating digits in the larger base. That is, a $2kt$ digit undulating number in base n will form a $2t$ digit undulating number in base n^k. In the bases I examined, this only affects bases 2 and 8, resulting in an infinite number of solutions of the form $(1010...)_2 = (2525...)_8$ and $(1010...)_2 = (5252...)_8$.

Likewise, some pairs of bases can be shown to have no solutions. For instance, bases n and n^k have no solutions for even k, since each group of k digits in the smaller base will form the same digit in the larger base, resulting in a repeated digit instead of an undulation. This shows there is no solutions for base pairs 2 and 4, 2 and 16, 3 and 9, or 4 and 16. This technique can be generalized to show, for instance, that the base pair 4 and 8 has no solution.

Although I checked numbers up to 400 digits long, I found no double smoothly undulating number larger than 959595, except for the infinite sequence of base 2 and 4 solutions. This suggests that there probably aren't any solutions for the pairs left unsolved in Table 1. Interesting future research would be to extend the results to additional bases, and to prove or disprove nonexistence of solutions for the base pairs without known solutions.

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Reference

1. C. A. Pickover, Is There a Double Smoothly Undulating Integer?, *Journal of Recreational Mathematics*, 22:1, pp. 52-53, 1990.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	-	y	n	?	?	y	y	?	?	?	y	y	?	?	n
3	y	-	y	y	?	y	y	n	y	y	?	?	y	y	?
4	n	y	-	y	y	y	n	y	?	y	?	y	?	y	n
5	?	y	y	-	y	y	y	y	y	y	y	?	y	y	y
6	?	?	y	y	-	y	y	y	y	y	y	y	y	y	y
7	y	y	y	y	y	-	y	y	y	y	y	y	y	y	y
8	y	y	n	y	y	y	-	y	y	y	y	y	y	y	n
9	?	n	y	y	y	y	y	-	y	y	y	y	y	y	y
10	?	y	?	y	y	y	y	y	-	y	y	y	y	y	y
11	?	y	y	y	y	y	y	y	y	-	y	y	y	y	y
12	y	?	?	y	y	y	y	y	y	y	-	y	y	y	y
13	y	?	y	?	y	y	y	y	y	y	y	-	y	y	y
14	?	y	?	y	y	y	y	y	y	y	y	y	-	y	y
15	?	y	y	y	y	y	y	y	y	y	y	y	y	-	y
16	n	?	n	y	y	y	n	y	y	y	y	y	y	y	-

Table 1. Pairs of bases with double smoothly undulating integers. A ‘y’ indicates a solution was found for the pair of bases. A ‘n’ indicates a solution does not exist for the pair of bases. A ‘?’ indicates no solution was found, but I haven’t proven nonexistence.