All Known Replicating Fibonacci Digits of 14 Digits or Fewer

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This paper gives eight replicating Fibonacci digits (repfigits) between 10 and 14 digits long. These numbers are believed to be previously unknown, as they exceed those listed by Pickover in [1]. This paper also describes two simple techniques that improved the search time from weeks to minutes and may be useful for solving similar problems in recreational mathematics.

The concept of repfigits was introduced in [2] by Keith. Given a n digit number D, first generate the Fibonacci sequence which starts with the n digits and has each successive term the sum of the n previous terms. The number D is called a repfigit if D appears in the resulting sequence. For example, 197 is a repfigit, since 197 is in the Fibonacci sequence (1, 9, 7, 17, 33, 57, 107, 197, 361, ...).

Table 1 lists all known repfigits of 14 digits or fewer. The 11 to 14 digit numbers are new results. There apparently are no 10 digit repfigits. The 2 to 9 digit results are from [1]. (Note that 2580 was mistyped in [1].)

The first technique to speed up the search changes the Fibonacci sequence search to a linear Diophantine equation. Note that the terms in a Fibonacci sequence are a linear combination of the original terms. For instance the sequence starting with 2 terms a, b is (a, b, $a+b$, $a+2b$, $2a+3b$, $3a+5b$, $5a+8b$, ...). Now suppose we suspect there is a 2 digit repfigit $10a+b$ that appears as the fifth term in the Fibonacci sequence. This means that $2a+3b = 10a+b$, so $8a-2b = 0$. Solving this equation for integer solutions between 0 and 9 yields solutions $(0,0)$, $(1,4)$ and $(2,8)$. Discarding the trivial solution, these correspond to the repfigits 14 and 28. Thus, to find n digit repfigits, we first generate the symbolic equations for the n digit Fibonacci sequence and determine what terms could have repfigits. (There are about 8 such terms for each n.) Then we solve the resulting linear Diophantine equations, restricting the values to the range 0 to 9.

For large numbers of digits, solving the Diophantine equation by trying all possible solutions is very slow. This search can be drastically sped up by breaking the equation into two parts and solving the parts separately. First, all possible values of the first part are computed and stored in a hash table. Then each value of the second part is computed and the hash table is checked to see if there is a solution for the appropriate value for the first part. As a simple example, consider solving the equation $4a+6b+3c+2d=0$, where a, b, c, and d are digits. With brute force, 10000 values would have to be tried. However, we can split the equation in half and enumerate the 100 values of $4a+6b$. Then, for each of the 100 values for $3c+2d$, we can immediately check if we found a way to make $-(3c+2d)$ with the first part. Thus, 200 computations are needed instead of 10000. By using this method, I could find solutions much faster than previous techniques; for 9 digit numbers, my search was 30000 times faster than the search described in [1]. Finding the 13 digit repfigits took about 30 minutes on a Sun SPARCstation 1 and finding the 14 digit repfigits took 10 1/2 hours.

Acknowledgements

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References

1. C. A. Pickover, All Known Replicating Fibonacci Digits less than One Billion, *Journal of Recreational Mathematics*, *22*:3, pp. 176-178, 1990.

2. M. Keith, Repfigit Numbers, *Journal of Recreational Mathematics, 19*:1, pp. 41-42, 1987.

Length	Replicating Fibonacci Digits
2	14, 19, 28, 47, 61, 75
3	197, 742
4	1104, 1537, 2208, 2580, 3684, 4788, 7385, 7647, 7909
5	31331, 34285, 34348, 55604, 62662, 86935, 93993
6	120284, 129106, 147640, 156146, 174680, 183186, 298320, 355419 694280, 925993
7	1084051, 7913837
8	11436171, 33445755, 44121607
9	129572008, 251133297
10	-none found-
11	24769286411, 96189170155
12	171570159070, 202366307758, 239143607789, 296658839738
13	1934197506555, 8756963649152
14	43520999798747, 74596893730427, 97295849958669

Table 1. All known replicating Fibonacci digits of 14 digits or fewer.